

Pomeron in the $\mathcal{N} = 4$ SYM at strong couplings

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Abstract. We show the result for the BFKL Pomeron intercept at $\mathcal{N} = 4$ Supersymmetric Yang-Mills model in the form of the inverse coupling expansion $j_0 = 2 - 2\lambda^{-1/2} - \lambda^{-1} + 1/4 \lambda^{-3/2} + 2(1 + 3\zeta_3)\lambda^{-2} + O(\lambda^{-5/2})$, which has been calculated recently in [1] with the use of the AdS/CFT correspondence.

1. Introduction

The investigation of the high energy behavior of scattering amplitudes in the $\mathcal{N} = 4$ Supersymmetric Yang-Mills (SYM) model [2]-[?] is important for our understanding of the Regge processes in QCD. Indeed, this conformal model can be considered as a simplified version of QCD, in which the next-to-leading order (NLO) corrections [8, 9] to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [10]-[14] are comparatively simple and numerically small. In the $\mathcal{N} = 4$ SYM the equations for composite states of several reggeized gluons and for anomalous dimensions (AD) of quasi-partonic operators turn out to be integrable at the leading logarithmic approximation [15, 16, 17]. Further, the eigenvalue of the BFKL kernel for this model has the remarkable property of the maximal transcendentality [3]. This property gave a possibility to calculate the AD γ of the twist-2 Wilson operators in one [18, 19], two [3, 20], three [21], four [22, 23] and five [24] loops using the QCD results [25] and the asymptotic Bethe ansatz [26] improved with wrapping corrections [23] in an agreement with the BFKL predictions [2, 3].

On the other hand, due to the AdS/CFT-correspondence [28, 29, 30], in $\mathcal{N} = 4$ SYM some physical quantities can be also computed at large couplings. In particular, for AD of the large spin operators Beisert, Eden and Staudacher constructed the integral equation [31] with the use of the asymptotic Bethe-ansatz. This equation reproduced the known results at small coupling constants and is in a full agreement (see [32, 33, 34]) with large coupling predictions [35, 36, 37].

With the use of the BFKL equation in a diffusion approximation [2, 5], strong coupling results for AD [35, 36, 37] and the pomeron-graviton duality [38, 39] the Pomeron intercept was calculated at the leading order in the inverse coupling constant (see the Erratum[40] to the paper [21]). Similar results were obtained also in Ref. [41]. The Pomeron-graviton duality in the $\mathcal{N} = 4$ SYM gives a possibility to construct the Pomeron interaction model as a generally covariant effective theory for the reggeized gravitons [42].

Below we present the strong coupling corrections to the Pomeron intercept $j_0 = 2 - \Delta$ in next orders. These corrections were obtained in Ref. [1] with the use of the recent calculations [43]-[47] of string energies.

2. BFKL equation at small coupling constant

The eigenvalue of the BFKL equation in $\mathcal{N} = 4$ SYM model has the following perturbative expansion [2, 3] (see also Ref. [5])

$$j - 1 = \omega = \frac{\lambda}{4\pi^2} \left[\chi(\gamma_{BFKL}) + \delta(\gamma_{BFKL}) \frac{\lambda}{16\pi^2} \right], \quad \lambda = g^2 N_c, \quad (1)$$

where λ is the t'Hooft coupling constant. The quantities χ and δ are functions of the conformal weights m and \tilde{m} of the principal series of unitary Möbius group representations, but for the conformal spin $n = m - \tilde{m} = 0$ they depend only on the BFKL anomalous dimension

$$\gamma_{BFKL} = \frac{m + \tilde{m}}{2} = \frac{1}{2} + i\nu \quad (2)$$

and are presented below [2, 3]

$$\chi(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1 - \gamma), \quad (3)$$

$$\delta(\gamma) = \Psi''(\gamma) + \Psi''(1 - \gamma) + 6\zeta_3 - 2\zeta_2\chi(\gamma) - 2\Phi(\gamma) - 2\Phi(1 - \gamma). \quad (4)$$

Here $\Psi(z)$ and $\Psi'(z)$, $\Psi''(z)$ are the Euler Ψ -function and its derivatives. The function $\Phi(\gamma)$ is defined as follows

$$\Phi(\gamma) = 2 \sum_{k=0}^{\infty} \frac{1}{k + \gamma} \beta'(k + 1), \quad \beta'(z) = \frac{1}{4} \left[\Psi'\left(\frac{z+1}{2}\right) - \Psi'\left(\frac{z}{2}\right) \right]. \quad (5)$$

Due to the symmetry of ω to the substitution $\gamma_{BFKL} \rightarrow 1 - \gamma_{BFKL}$ expression (1) is an even function of ν

$$\omega = \omega_0 + \sum_{m=1}^{\infty} (-1)^m D_m \nu^{2m}, \quad (6)$$

where

$$\omega_0 = 4 \ln 2 \frac{\lambda}{4\pi^2} \left[1 - \bar{c}_1 \frac{\lambda}{16\pi^2} \right] + O(\lambda^3), \quad (7)$$

$$D_m = 2 \left(2^{2m+1} - 1 \right) \zeta_{2m+1} \frac{\lambda}{4\pi^2} + \frac{\delta^{(2m)}(1/2)}{(2m)!} \frac{\lambda^2}{64\pi^4} + O(\lambda^3). \quad (8)$$

According to Ref. [3] we have

$$\bar{c}_1 = 2\zeta_2 + \frac{1}{2 \ln 2} \left(11\zeta_3 - 32\text{Ls}_3\left(\frac{\pi}{2}\right) - 14\pi\zeta_2 \right) \approx 7.5812, \quad \text{Ls}_3(x) = - \int_0^x \ln^2 \left| 2 \sin\left(\frac{y}{2}\right) \right| dy. \quad (9)$$

Due to the Möbius invariance and hermicity of the BFKL hamiltonian in $\mathcal{N} = 4$ SYM expansion (6) is valid also at large coupling constants. In the framework of the AdS/CFT correspondence the BFKL Pomeron is equivalent to the reggeized graviton [39]. In particular, in the strong coupling regime $\lambda \rightarrow \infty$

$$j_0 = 2 - \Delta, \quad (10)$$

where the leading contribution $\Delta = 2/\sqrt{\lambda}$ was calculated in Refs. [40, 41]. Below we find NLO terms in the strong coupling expansion of the Pomeron intercept.

3. AdS/CFT correspondence

Due to the energy-momentum conservation, the universal AD of the stress tensor $T_{\mu\nu}$ should be zero, i.e.,

$$\gamma(j=2) = 0. \quad (11)$$

It is important, that the AD γ contributing to the DGLAP equation [48]-[52] does not coincide with γ_{BFKL} appearing in the BFKL equation. They are related as follows [8, 53, 54]

$$\gamma = \gamma_{BFKL} + \frac{\omega}{2} = \frac{j}{2} + i\nu, \quad (12)$$

where the additional contribution $\omega/2$ is responsible in particular for the cancelation of the singular terms $\sim 1/\gamma^3$ obtained from the NLO corrections (1) to the eigenvalue of the BFKL kernel [8]. Using above relations one obtains

$$\nu(j=2) = i. \quad (13)$$

As a result, from eq. (6) for the Pomeron trajectory we derive the following representation for the correction Δ (10) to the graviton spin 2

$$\Delta = \sum_{m=1}^{\infty} D_m. \quad (14)$$

According to (10) and (14), we have the following small- ν expansion for the eigenvalue of the BFKL kernel

$$j-2 = \sum_{m=1}^{\infty} D_m \left((-\nu^2)^m - 1 \right), \quad (15)$$

where ν^2 is related to γ according to eq. (12)

$$\nu^2 = -\left(\frac{j}{2} - \gamma \right)^2. \quad (16)$$

On the other hand, due to the ADS/CFT correspondence the string energies E in dimensionless units are related to the AD γ of the twist-two operators as follows [29, 30]¹

$$E^2 = (j + \Gamma)^2 - 4, \quad \Gamma = -2\gamma \quad (17)$$

and therefore we can obtain from (16) the relation between the parameter ν for the principal series of unitary representations of the Möbius group and the string energy E

$$\nu^2 = -\left(\frac{E^2}{4} + 1 \right). \quad (18)$$

This expression for ν^2 can be inserted in the r.h.s. of Eq. (15) leading to the following expression for the Regge trajectory of the graviton in the anti-de-Sitter space

$$j-2 = \sum_{m=1}^{\infty} D_m \left[\left(\frac{E^2}{4} + 1 \right)^m - 1 \right]. \quad (19)$$

¹ Note that our expression (17) for the string energy E differs from a definition, in which E is equal to the scaling dimension Δ_{sc} . But eq. (17) is correct, because it can be presented as $E^2 = (\Delta_{sc} - 2)^2 - 4$ and coincides with Eqs. (45) and (3.44) from Refs. [29] and [30], respectively.

4. Graviton Regge trajectory and Pomeron intercept

We assume, that eq. (19) is valid also at large j and large λ in the region $1 \ll j \ll \sqrt{\lambda}$, where the strong coupling calculations of energies were performed [43, 47]. These energies can be presented in the form ²

$$\frac{E^2}{4} = \sqrt{\lambda} \frac{S}{2} \left[h_0(\lambda) + h_1(\lambda) \frac{S}{\sqrt{\lambda}} + h_2(\lambda) \frac{S^2}{\lambda} \right] + O(S^{7/2}), \quad (20)$$

where

$$h_i(\lambda) = a_{i0} + \frac{a_{i1}}{\sqrt{\lambda}} + \frac{a_{i2}}{\lambda} + \frac{a_{i3}}{\sqrt{\lambda^3}} + \frac{a_{i2}}{\lambda^2}. \quad (21)$$

The contribution $\sim \sqrt{S}$ can be extracted directly from the Basso result [44, 45] taking $J_{an} = 2$ according to [46]:

$$h_0(\lambda) = \frac{I_3(\sqrt{\lambda})}{I_2(\sqrt{\lambda})} + \frac{2}{\sqrt{\lambda}} = \frac{I_1(\sqrt{\lambda})}{I_2(\sqrt{\lambda})} - \frac{2}{\sqrt{\lambda}}, \quad (22)$$

where $I_k(\sqrt{\lambda})$ is the modified Bessel functions. It leads to the following values of coefficients a_{0i}

$$a_{00} = 1, \quad a_{01} = -\frac{1}{2}, \quad a_{02} = a_{03} = \frac{15}{8}, \quad a_{04} = \frac{135}{128} \quad (23)$$

The coefficients a_{10} and a_{20} come from considerations of the classical part of the folded spinning string corresponding to the twist-two operators (see, for example, [47])

$$a_{10} = \frac{3}{4}, \quad a_{20} = -\frac{3}{16}. \quad (24)$$

The one-loop coefficient a_{11} is found recently in the paper [46], considering different asymptotical regimes with taking into account the Basso result [44] (ζ_3 is the Euler ζ -function)

$$a_{11} = \frac{3}{16} (1 - \zeta_3), \quad (25)$$

Comparing the l.h.s. and r.h.s. of (19) at large j values gives us the coefficients D_m and Δ (see Appendix A in [1]).

5. Conclusion

We have shown the intercept of the BFKL pomeron at weak coupling regime and demonstrated an approach to obtain its values at strong couplings (for details, see Ref. [1]).

At $\lambda \rightarrow \infty$, the correction Δ for the Pomeron intercept $j_0 = 2 - \Delta$ has the form ³

$$\Delta = \frac{2}{\lambda^{1/2}} \left[1 + \frac{1}{2\lambda^{1/2}} - \frac{1}{8\lambda} - \left(1 + 3\zeta_3 \right) \frac{1}{\lambda^{3/2}} + \left(2a_{12} - \frac{145}{128} - \frac{9}{2}\zeta_3 \right) \frac{1}{\lambda^2} + O\left(\frac{1}{\lambda^{5/2}} \right) \right]. \quad (26)$$

The fourth corrections in (26) contain unknown coefficient a_{12} , which will be obtained after the evaluation of spinning folded string on the two-loop level. Some estimations were given in Section 6 of [1].

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² Here we put $S = j - 2$, which in particular is related to the use of the angular momentum $J_{an} = 2$ in calculations of Refs [43, 47].

³ Using a similar approach, the coefficients $\sim \lambda^{-1}$ and $\sim \lambda^{-3/2}$ were calculated also in the paper [55]. After correction of some errors, the results in [55] coincide with ours.

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